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ANALYTICAL PHENOMENON

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Chapter 0
FAMOUS THEOREMS

CAUCHY–SCHWARZ Inequality

$$(ax+by)^2 \le (a^2+b^2)(x^2+y^2);\ a,b,x,y \in \mathbb{R}$$

$$(ax+by+cz)^2 \le (a^2+b^2+c^2)(x^2+y^2+z^2);\ a,b,c,x,y,z \in \mathbb{R}$$

$$\left(\sum_{i=1}^{n} a_i x_i\right)^2 \le \left(\sum_{i=1}^{n} a_i^2\right)\left(\sum_{i=1}^{n} x_i^2\right);\ a_i, x_i \in \mathbb{R}, i \in \overline{1,n}$$

$$\frac{a^2}{x}+\frac{b^2}{y} \ge \frac{(a+b)^2}{x+y};\ a,b \in \mathbb{R};\ x,y \in (0,\infty)$$

$$\frac{a^2}{x}+\frac{b^2}{y}+\frac{c^2}{z} \ge \frac{(a+b+c)^2}{x+y+z};\ a,b,c \in \mathbb{R};\ x,y,z \in (0,\infty)$$

$$\frac{a}{x}+\frac{b}{y}+\frac{c}{z} \ge \frac{(a+b+c)^2}{ax+by+cz};\ a,b,c,x,y,z \in (0,\infty)$$

$$\frac{a_1^2}{x_1}+\frac{a_2^2}{x_2}+\cdots+\frac{a_n^2}{x_n} \ge \frac{(a_1+a_2+\cdots+a_n)^2}{x_1+x_2+\cdots+x_n};\ a_i \in \mathbb{R};\ x_i > 0;\ i \in \overline{1,n}$$

$$\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b} \ge \frac{3}{2};\ (\forall)a,b,c \in (0,\infty)$$

$$\frac{a}{b+nc}+\frac{b}{c+na}+\frac{c}{a+nb} \ge \frac{3}{n+1};\ a,b,c \in (0,\infty);\ n \in \mathbb{N}^*$$

MINKOWSKI's Inequality

$$\sqrt{(x+a)^2+(y+b)^2} \le \sqrt{x^2+y^2}+\sqrt{a^2+b^2}$$

$$\sqrt{(x+y+z)^2+(a+b+c)^2} \le \sqrt{x^2+a^2}+\sqrt{y^2+b^2}+\sqrt{z^2+c^2}$$

$$\sqrt{(x+a)^2+(y+b)^2+(z+c)^2} \le \sqrt{x^2+y^2+z^2}+\sqrt{a^2+b^2+c^2}$$

$$\sqrt{(x_1+a_1)^2+(x_2+a_2)^2+\cdots+(x_n+a_n)^2} \le \sqrt{x_1^2+x_2^2+\cdots+x_n^2}+\sqrt{a_1^2+a_2^2+\cdots+a_n^2}$$

$$x_i; a_i \in \mathbb{R};\ i \in \overline{1,n};\ n \in \mathbb{N}^*$$

$$\left(\sum_{i=1}^{n} |x_i+y_i|^p\right)^{\frac{1}{p}} \le \left(\sum_{i=1}^{n} |x_i|^p\right)^{\frac{1}{p}} + \left(\sum_{i=1}^{n} |y_i|^p\right)^{\frac{1}{p}}$$

$$p > 1;\ x_i, y_i \in \mathbb{R};\ i \in \overline{1,n};\ n \in \mathbb{N}^*$$

HÖLDER's Inequality

$$\frac{a^3}{x}+\frac{b^3}{y}+\frac{c^3}{z} \ge \frac{(a+b+c)^3}{3(x+y+z)};\ a,b,c,x,y,z \in (0,\infty)$$

$$\frac{a^4}{x}+\frac{b^4}{y}+\frac{c^4}{z}\geq\frac{(a+b+c)^4}{9(x+y+z)};a,b,c,x,y,z\in(0,\infty)$$

$$\frac{a^n}{x}+\frac{b^n}{y}+\frac{c^n}{z}\geq\frac{(a+b+c)^n}{3^{n-2}(x+y+z)};a,b,c,x,y,z\in(0,\infty);n\in\mathbb{N};n\geq 2$$

$$\frac{a^n}{x}+\frac{b^n}{y}\geq\frac{(a+b)^n}{2^{n-2}(x+y)};a,b,x,y\in(0,\infty);\ n\geq 2;n\in\mathbb{N}$$

$$\frac{x^3}{a^2}+\frac{y^3}{b^2}+\frac{z^3}{c^2}\geq\frac{(x+y+z)^3}{(a+b+c)^2};x,y,z,a,b,c\in(0,\infty)$$

$$\frac{x^4}{a^3}+\frac{y^4}{b^3}+\frac{z^4}{c^3}\geq\frac{(x+y+z)^4}{(a+b+c)^3};x,y,z,a,b,c\in(0,\infty)$$

$$\frac{x^{n+1}}{a^n}+\frac{y^{n+1}}{b^n}+\frac{z^{n+1}}{c^n}\geq\frac{(x+y+z)^{n+1}}{(a+b+c)^n};x,y,z,a,b,c\in(0,\infty);\ n\in\mathbb{N}$$

$$\left(\sum_{i=1}^{n}a_i^3\right)\left(\sum_{i=1}^{n}b_i^3\right)\left(\sum_{i=1}^{n}c_i^3\right)\geq\left(\sum_{i=1}^{n}a_ib_ic_i\right)^3;a_i,b_i,c_i\in[0,\infty);n\in\mathbb{N}^*$$

$$\left(\sum_{i=1}^{n}a_i^4\right)\left(\sum_{i=1}^{n}b_i^4\right)\left(\sum_{i=1}^{n}c_i^4\right)\left(\sum_{i=1}^{n}d_i^4\right)\geq\left(\sum_{i=1}^{n}a_ib_ic_id_i\right)^4;a_i,b_i,c_i,d_i\in\mathbb{R};n\in\mathbb{N}^*$$

$$\sum_{i=1}^{n}|x_iy_i|\leq\left(\sum_{i=1}^{n}|x_i|^p\right)^{\frac{1}{p}}\cdot\left(\sum_{i=1}^{n}|y_i|^q\right)^{\frac{1}{q}};p,q\in(1,\infty)$$

$$\frac{1}{p}+\frac{1}{q}=1;x_i,y_i\in\mathbb{R};i\in\overline{1,n};n\in\mathbb{N}^*$$

HUYGENS's Inequality

$$(1+a_1)(1+a_2)\geq\left(1+\sqrt{a_1a_2}\right)^2;a_1,a_2\in[0,\infty)$$

$$(1+a_1)(1+a_2)(1+a_3)\geq\left(1+\sqrt[3]{a_1a_2a_3}\right)^3;a_1,a_2,a_3\in[0,\infty)$$

$$(1+a_1)(1+a_2)(1+a_3)(1+a_4)\geq\left(1+\sqrt[4]{a_1a_2a_3a_4}\right)^4;a_1,a_2,a_3,a_4\in[0,\infty)$$

$$\prod_{i=1}^{n}(1+x_i)\geq\left(1+\sqrt[n]{x_1x_2\cdots x_n}\right)^n;a_i\in[0,\infty);\ n\in\mathbb{N};n\geq 2$$

$$(a_1+b_1)(a_2+b_2)\geq\left(\sqrt{a_1a_2}+\sqrt{b_1b_2}\right)^2$$

$$(a_1+b_1)(a_2+b_2)(a_3+b_3)\geq\left(\sqrt[3]{a_1a_2a_3}+\sqrt[3]{b_1b_2b_3}\right)^3$$

$$(a_1+b_1)(a_2+b_2)\cdot\ldots\cdot(a_n+b_n)\geq\left(\sqrt[n]{a_1a_2\cdot\ldots\cdot a_n}+\sqrt[n]{b_1b_2\cdot\ldots\cdot b_n}\right)^n$$

$$(a_1+b_1+c_1)(a_2+b_2+c_2)\geq\left(\sqrt{a_1a_2}+\sqrt{b_1b_2}+\sqrt{c_1c_2}\right)^2$$

$$(a_1+b_1+c_1)(a_2+b_2+c_2)\cdot\ldots\cdot(a_n+b_n+c_n)$$
$$\geq\left(\sqrt[n]{a_1a_2\cdot\ldots\cdot a_n}+\sqrt[n]{b_1b_2\cdot\ldots\cdot b_n}+\sqrt[n]{c_1c_2\cdot\ldots\cdot c_n}\right)^n$$

Generalization of HÖLDER's Inequality

$$\prod_{i=1}^{n}\left(\sum_{j=1}^{n} x_{ij}\right)^{w_j} \geq \sum_{i=1}^{n}\left(\prod_{j=1}^{n} x_{ij}^{w_i}\right)$$
$$w_1 + w_2 + \cdots + w_n = 1$$

CHEBYSHEV's Inequality

$$\begin{cases} (x_1 \leq x_2) \wedge (y_1 \leq y_2) \text{ or } (x_1 \geq x_2) \wedge (y_1 \geq y_2) \\ x_1y_1 + x_2y_2 \geq \frac{1}{2}(x_1 + x_2)(y_1 + y_2) \end{cases}$$

$$\begin{cases} (x_1 \leq x_2 \leq x_3) \wedge (y_1 \leq y_2 \leq y_3) \text{ or } (x_1 \geq x_2 \geq x_3) \wedge (y_1 \geq y_2 \geq y_3) \\ x_1y_1 + x_2y_2 + x_3y_3 \geq \frac{1}{3}(x_1 + x_2 + x_3)(y_1 + y_2 + y_3) \end{cases}$$

$$\begin{cases} (x_1 \leq x_2 \leq \cdots \leq x_n) \wedge (y_1 \leq y_2 \leq \cdots \leq y_n) \text{ or } (x_1 \geq x_2 \geq \cdots \geq x_n) \wedge (y_1 \geq y_2 \geq \cdots \geq y_n) \\ x_1y_1 + x_2y_2 + \cdots + x_ny_n \geq \frac{1}{n}(x_1 + x_2 + \cdots + x_n)(y_1 + y_2 + \cdots + y_n) \end{cases}$$

$$\begin{cases} (x_1 \leq x_2) \wedge (y_1 \geq y_2) \text{ or } (x_1 \geq x_2) \wedge (y_2 \leq y_2) \\ x_1y_1 + x_2y_2 \leq \frac{1}{2}(x_1 + x_2)(y_1 + y_2) \end{cases}$$

$$\begin{cases} (x_1 \leq x_2 \leq x_3) \wedge (y_1 \geq y_2 \geq y_3) \text{ or } (x_1 \geq x_2 \geq x_3) \wedge (y_1 + y_2 + y_3) \\ x_1y_1 + x_2y_2 + x_3y_3 \leq \frac{1}{3}(x_1 + x_2 + x_3)(y_1 + y_2 + y_3) \end{cases}$$

$$\begin{cases} (x_1 \leq x_2 \leq \cdots \leq x_n) \wedge (y_1 \geq y_2 \geq \cdots \geq y_n) \text{ or } (x_1 \geq x_2 \geq \cdots \geq x_n) \wedge \\ \wedge (y_1 \leq y_2 \leq \cdots \leq y_n) \\ x_1y_1 + x_2y_2 + \cdots + x_ny_n \leq \frac{1}{n}(x_1 + x_2 + \cdots + x_n)(y_1 + y_2 + \cdots + y_n) \\ x_i, y_i \in \mathbb{R}; n \in \mathbb{N}^*; i \in \overline{1,n} \end{cases}$$

$$\begin{cases} \sum_{i=1}^{n} f(a_i)g(b_i)p_i \geq \left(\sum_{i=1}^{n} f(a_i)p_i\right)\left(\sum_{i=1}^{n} g(b_i)p_i\right) \geq \sum_{i=1}^{n} f(a_i)g(b_{n-i+1})\,p_i \\ \text{for } a_1 \leq a_2 \leq \cdots \leq a_n; b_1 \leq b_2 \leq \cdots \leq b_n \\ p_i \geq 0; i \in \overline{1,n}; n \in \mathbb{N}^*; p_1 + p_2 + \cdots + p_n = 1 \end{cases}$$

$$f, g \text{ non-decreasing}$$